Joint User Association and Resource Allocation in Small Cell Networks with Backhaul Constraints

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***Abstract*—Heterogeneous networks potentially provide significant capacity gains by overlaying the traditional cellular network with a layer of small cells (SCs) served by access points (APs). However, the limited backhaul capacity of the SC APs, combined with increased interference from neighboring cells, necessitates careful resource allocation to realize the gains. In our work, we consider maximization of the weighted sum rate in small cell networks with carrier aggregation while enforcing a backhaul constraint on each SC AP. We propose an efficient, waterfilling-like, algorithm which converges to a locally optimal solution of the non-convex optimization problem. This algorithm differs from existing works by using a computationally efficient bisection-like search that ensures the sum-power and sum-rate constraints at each AP are satisfied via an one-dimensional search. An added advantage is that this approach also allows for a decentralized implementation.**

I. Introduction

Based on dense deployments of access points, small cell (SC) networks have shown great promise in meeting the rapid growth in demand for mobile data [1], [2]. SC networks augment the traditional cellular network by providing efficient frequency reuse through decreasing distance between mobile user equipment (UE) and the access point (AP). However with transmissions from closely-spaced APs, interference needs to be properly addressed and controlled to reap these benefits [3], [4]. Furthermore, while traditional base stations (BSs) are connected to the core network through a large capacity/low delay backhaul link, SCs usually do not have that luxury: for example femtocells are connected to the core network via a internet connection which has significantly less bandwidth and increased delay [5], [6].

In a parallel development, Coordinated Multipoint (CoMP) has been proposed to manage inter-cell interference and increase network-wide data rates [7]. CoMP refers to multiple BSs coordinating transmissions to individual users via beamforming and resource allocation. CoMP, though, assumes that the data for every user is available at every BS. In considering the use of coordinated transmissions for SC networks, the limited backhaul available becomes a crucial limitation. The effect of backhaul constraints has been studied extensively in the literature but mostly in the context of the cost of signaling overhead that needs to be exchanged between cooperative nodes: in both [8] and [9] downlink CoMP algorithms were proposed that reduce the signaling needed.

Motivated by the need to manage interference and increase data rates, this paper extends CoMP to dense SC networks. We maximize the sum rate to a group of users for orthogonal frequency division multiple access (OFDMA) networks that allow for carrier aggregation while imposing a backhaul constraint. Backhaul constraints with CoMP have been considered before, e.g., capacity bounds and a rate/backhaul tradeoff in different scenarios were studied in [10] which showed that significant performance gains were possible even with a strongly limited backhaul. In [11], the authors use stochastic geometry to quantify the signaling overhead involved in cooperation. The effect of backhaul delay was investigated in [12] & [13], an issue not considered here.

Carrier aggregation allows for a user to be served by different basestations on different sub-carriers [14], [15]. The work in [16] showed that this provides significant gains over restricting a user to be served by a single base station. There are several works that deal with the effect of per cell sum rate constraints. In [17], an effective heuristic algorithm is proposed for dynamic link selection in a downlink OFDMA network that considers both the sum rate constraints and overhead constraints between cooperating cells. The spectrum allocation, user scheduling, power control and backhaul feasibility portions of the algorithm are done separately. In [18] extend this to the multiple-antenna case.

An alternative backhaul architecture was studied in [19] where wireless backhaul nodes for picocells was studied, again the backhaul scheduling and power control steps are done separately. A decentralized algorithm for the multi-antenna scenario was studied in [20] where the algorithm uses a two-dimensional search to obtain the dual parameters corresponding with power and backhaul feasibility at each base-station. The authors of [21] provide a framework for deriving the upper bound on the utility in backhaul limited networks and augmented Lagrangian method based algorithm for near optimal performance.

In this work we analyze the weighted-sum user rate maximization problem in the downlink for backhaul constrained single antenna OFDMA networks with carrier aggregation and propose a computationally efficient decentralized algorithm based on the approach of [22]. Our novel approach jointly performs spectrum allocation, user scheduling, cell association, power control and backhaul optimization with a bisection search approach to determining the dual variables to achieve both sum rate and sum power feasibility. This provides significantly simpler implementation and computational complexity over traditional subgradient and augmented Lagrangian approaches. The proposed algorithm solves the equations imposed by the Karush-Kuhn-Tucker (KKT) system directly and converges to a locally optimal solution.

II. System Model

We consider the downlink of an OFDMA system with N orthogonal subcarriers, M randomly distributed APs, and K randomly distributed users. Each AP m is subject to a sum power constraint $P_{max,m}$ and a sum rate constraint $B_{max,m}$ over all subchannels and users. The APs and users are distributed uniformly distribution throughout a geographical square area with side length of d_{area}.

The channel gain between AP m and user k on subchannel n is denoted by $G_{mkn} = |H_{mkn}|^2$ where H_{mkn} represents the respective complex channel. We assume a block flat-fading model on each subcarrier where the channel gain is constant. Additionally, we assume perfect and instantaneous channel state information (CSI) in this paper and also ignore the delay and overhead needed to synchronize CSI between cells.

The channel model accounts for path loss and log-normal fading. The path loss in dB between the k^{th} user and m^{th} AP is given by the 3GPP model [23]:

$$\begin{aligned}\gamma_{mk} =& \max\left(15.3 + 37.6\log_{10} d_{mk}, 37 + 20\log_{10} d_{mk}\right) \\ &+ q_{mk}W + L,\end{aligned} \tag{1}$$

where d_{mk} is the distance between user k and AP m in meters, q_{mk} is a random variable representing the total number of internal walls between user k and AP m, W is the partition loss of internal walls in dB, L is the penetration loss of an outdoor wall in dB. Now, $h_{mkn} \sim \mathcal{CN}(0,1)$ represents the small-scale fading between user k and AP m on subchannel n. The overall channel gain between user k and AP m on subchannel n is given by

$$G_{mkn,\ dB} = \gamma_{mk} + 10\log_{10}|h_{mkn}|^2 + \psi, \tag{2}$$

where $\psi \sim LN(\mu_s, \sigma_s)$ represents log-normal shadowing.

For simplicity, the rate R_{mkn} in (bits/Hz) for the transmission on subchannel n from base station m to user k is the determined by the Signal-to-Interference ratio (SINR) at the receiver (user):

$$R_{mkn} = \log_2\left(1 + \frac{P_{mkn}G_{mkn}}{I_{mkn} + \sigma^2}\right), \tag{3}$$

where P_{mkn} is the transmit power for AP m to user k on subchannel n and I_{mkn} denotes the interference at the user and σ^2 is the additive white Gaussian noise power. Further, the interference at the user is due to other transmissions by the same AP and transmissions from other APs, i.e.,

$$I_{mkn} = \sum_{k'=1,\ k'\neq k}^{K} P_{mk'n}G_{mkn} + \sum_{m'=0,\ m'\neq m}^{M}\sum_{k''=0}^{K} P_{m'k''n}G_{m'kn} \tag{4}$$

Carrier aggregation (CA) allows each user to combine data streams from different sources on the orthogonal subchannels. This allows us to exploit user diversity and balance transmission loads across APs in the system. In a system limited by a backhaul constraint, this flexibility is particularly useful. In such a system, the aggregate rate achieved by user k is:

$$R_k = \sum_{m,n} R_{mkn}, \tag{5}$$

which is the sum of rates achieved over all subchannels from all APs.

III. Problem Formulation

We consider resource allocation to maximize the weighted sum-rate (WSR) of all users in the downlink transmission:

$$\begin{aligned}\text{maximize}\ & \sum_{k=1}^{K}\alpha_k R_k & (6)\\ \text{subject to}\ & P_{total,m} = \sum_{k,n} P_{mkn} \leq P_{max,m}, \forall\ m, & (7)\\ & R_{total,m} = \sum_{k,n} R_{mkn} \leq B_{max,m}, \forall\ m, & (8)\\ & P_{mkn} \geq 0, \forall m,k,n. & (9)\end{aligned}$$

Here $\alpha_k \geq 0$ are the user-specific weights, (7) represents the usual sum-power constraint, (8) the backhaul constraint while (9) enforces that the power used is always non-negative. When the power $P_{mkn} > 0$, base station m will transmit over subchannel n user k. Users which do not receive any power are not scheduled in that given time-slot. The constraint that a AP cannot serve two users on the same channel is not included explicitly in the problem formulation but is automatically enforced by the algorithm since such a situation is sub-optimal. Carrier aggregation removes the constraint that each user must be served by only one AP which allows for dynamic user association. In this way, user association, user scheduling, spectrum allocation and power control is solved in a joint manner.

Equations (6)-(9) represent the WSR optimization problem and solving this efficiently is the core contribution of this paper. The optimization problem is well known to be non-convex and here we expand on the Improved Iterative Waterfilling algorithm of [22]. One advantage of solving the weighted sum-rate problem is that by selecting appropriate weights α_k, we can introduce fairness measures such as proportional fairness $\alpha_k = 1/\bar{R}_k(T)$, where $\bar{R}_k(T)$ is the rate allocated to user k in the previous T time slots [24].

IV. Lagrangian and KKT Conditions

We form the Lagrangian of the objective function with the power and backhaul constraints. We denote as ν_m the dual variable for the backhaul constraint of AP m, λ_m as the dual variable for the power constraint of AP m. The vector of power allocations for all triples (m,k,n) is denoted as $\boldsymbol{P}$, also $\boldsymbol{\nu}$ and $\boldsymbol{\lambda}$ are the vectors for the respective dual variables.

$$\frac{\partial \mathcal{L}(\boldsymbol{P},\boldsymbol{\nu},\boldsymbol{\lambda})}{\partial P_{mkn}} = \frac{\alpha_k G_{mkn}}{P_{mkn}G_{mkn}+I_{mkn}+\sigma^2} - \sum_{k'\neq k}\frac{(\alpha_{k'}+\nu_m)\,G_{mk'n}}{I_{mk'n}+\sigma^2}\cdot\frac{P_{mk'n}G_{mk'n}}{P_{mk'n}G_{mk'n}+I_{mk'n}+\sigma^2}$$
$$-\sum_{m'\neq m}\sum_{k'}\frac{(\alpha_{k'}-\nu_{m'})\,G_{mk'n}}{I_{mk'n}+\sigma^2}\cdot\frac{P_{m'k'n}G_{m'k'n}}{P_{m'k'n}G_{m'k'n}+I_{m'k'n}+\sigma^2}-\lambda_m$$
$$= 0 \tag{10}$$

$$t_{mkn} = \sum_{k'\neq k}\frac{(\alpha_{k'}-\nu_m)\,G_{mk'n}}{I_{mk'n}+\sigma^2}\cdot\frac{P_{mk'n}G_{mk'n}}{P_{mk'n}G_{mk'n}+I_{mk'n}+\sigma^2} + \sum_{m'\neq m}\sum_{k'}\frac{(\alpha_{k'}+\nu_{m'})\,G_{mk'n}}{I_{m'k'n}+\sigma^2}\cdot\frac{P_{m'k'n}G_{m'k'n}}{P_{m'k'n}G_{m'k'n}+I_{m'k'n}+\sigma^2} \tag{11}$$

$$\mathcal{L}(\boldsymbol{P},\boldsymbol{\nu},\boldsymbol{\lambda}) = \sum_{k=1}^{K}\alpha_k\sum_{\forall m,n}\log\left(1+\frac{P_{mkn}G_{mkn}}{I_{mkn}+\sigma^2}\right) + \sum_{m=1}^{M}\nu_m\left(B_{max,m}-\sum_{\forall k,n}R_{mkn}\right) + \sum_{m=1}^{M}\lambda_m\left(P_{max,m}-\sum_{\forall k,n}P_{mkn}\right) \tag{12}$$

By substituting the explicit equations for rates, we can analyze the KKT conditions for local optimality. Taking the partial derivative of the Lagrangian with respect to a specific power P_{mkn} and set it to 0 we get (10).

By rearranging the results, we arrive at the following expression:

$$\frac{\alpha_k-\nu_m}{P_{mkn}+\frac{I_{mkn}+\sigma^2}{G_{mkn}}} = \lambda_m + t_{mkn} \tag{13}$$

The term t_{mkn}, given in (11), summarizes the effect of the interference caused by user k being served by AP m on channel n on the other users and subchannels. Now we have an explicit expression to calculate the power P_{mkn}:

$$P_{mkn} = \left(\frac{\alpha_k-\nu_m}{t_{mkn}-\lambda_m}-\frac{I_{mkn}+\sigma^2}{G_{mkn}}\right)^+, \tag{14}$$

where $(x)^+$ denotes $\max(x,0)$.

By fixing I_{mkn} and t_{mkn}, the power P_{mkn} is completely dependent on the dual variables ν_m and λ_m. Intuitively, $\boldsymbol{\nu}$ takes values in $[0,\alpha_m]$, to penalize the power assigned by AP m when it is about to violate the backhaul constraint. Similarly, $\boldsymbol{\lambda}$ take values in $[0,\infty]$ which corresponds to the inverse of the water level. The term t_{mkn} can be interpreted as prices: the larger the effect of interference caused by P_{mkn}, the higher t_{mkn} which will lower the allocated power for AP m, if the price of using link (m,k,n) is too high, zero power will be allocated to that link.

This KKT system can be solved by fixing the interference of other links and calculate the appropriate dual variables $(\boldsymbol{\lambda},\boldsymbol{\nu})$ and P_{mkn}. We then update the terms t_{mkn} and I_{mkn} according to the new $\boldsymbol{P}$ and repeat this process until convergence. If AP m is not power constrained or backhaul constrained, its corresponding dual parameters will be zero ($\lambda_m=0$ and $\nu_m=0$). The feasibility of the sum power and sum rate constraints at each AP can be described by the following two equations:

$$P_{max,m} \geq \sum_{n,k}\left(\frac{\alpha_k-\nu_m}{t_{mkn}-\lambda_m}-\frac{I_{mkn}+\sigma^2}{G_{mkn}}\right)^+, \tag{15}$$

$$B_{max,m} \geq \sum_{n,k}\log\left(1+\frac{P_{mkn}G_{mkn}}{I_{mkn}+\sigma^2}\right). \tag{16}$$

Finding the optimal set of $\boldsymbol{\lambda}$ and $\boldsymbol{\nu}$ could be done by using a two-dimensional search or classic constrained optimization techniques such as subgradient method or augmented Lagrangian but this is computationally intensive and might not be practical for large problem sizes [25].

V. Proposed Algorithm

We note that both the sum power and sum rate constraints are functions of powers P_{mkn}. Fixing I_{mkn} and t_{mkn}, the power is a monotonic function of $\boldsymbol{\lambda}$ when $\boldsymbol{\nu}$ are fixed and vice versa. The rate R_{mkn} is also a monotonic function of power P_{mkn} when I_{mkn} is fixed and therefore is completely determined by the dual variables.

Complementary slackness in constrained optimization states that for inequality constraints $f_i(x)\leq 0$ that are tight with equality, the associated dual variable is non-zero [26]. Using this result, at a local optimum each base station could be either power constrained or rate constrained, so we can perform our search on a single dual variable via bisection instead of a two dimensional search.

Searching over the dual variable for power λ_m and setting $\nu_m=0$ with the termination condition that either the power-allocated is within ϵ of $P_{max,m}$ or the rate allocated is within ϵ of $B_{max,m}$. In the case that the base station is both power and backhaul constrained, the achieved rate will be the same as when searching over two dual variables.

The algorithm to solve the WSR problem is listed as Algorithm 1. The innovation is the bisection search as detailed in Algorithm 2. As mentioned earlier, it is strictly suboptimal for an AP to transmit to two users on the same subcarrier (the power allocated to the weaker user could be transferred to stronger user). However, since our solution is sub-optimal, to ensure that the final solution has each AP transmitting to one user on each subchannel, we incorporate the following rule after the waterfilling step: for AP $m = m_0$ only the user $k = k_0$ allocated with the most power $P_{m_0,k_0,n_0} = P^{max}_{m_0,n_0}$ is allowed to transmit, other transmit powers for users on the same subchannel $n = n_0$ are set to 0:

$$P_{m_0,k,n_0} = \begin{cases} P^{max}_{m_0,n_0}, & \text{if } k = k_0. \\ 0, & \text{otherwise.} \end{cases} \tag{17}$$

Algorithm 1 Improved Iterative Water-filling Algorithm with Backhaul

Initialize $\boldsymbol{P}$, t_{mkn}
loop until t_{mkn} converges
    loop until $\boldsymbol{P}$ converges
        for AP $m = 1 \cdots M$ **do**
            Calculate I_{mkn} according to (4).
            Obtain λ_m via bisection search in Algorithm 2.
            Calculate $\boldsymbol{P}$ using (14).
            Update $\boldsymbol{P}$ according to (17).
        end for
    end loop
    Update t_{mkn} according to (11).
end loop

Algorithm 2 Bisection Search of λ_m

Fix t_{mkn} and I_{mkn}.
Initialize $\lambda_{m,min}$, $\lambda_{m,max}$, λ_m.
loop until $B_{max,m} - R_{total,m} \leq \epsilon$ or $P_{max,m} - P_{total,m} \leq \epsilon$
    Calculate P_{mkn} from (14) and update $P_{total,m}$.
    Calculate R_{mkn} from (3) and update $R_{total,m}$.
    if $P_{total,m} > P_{max,m}$ or $R_{total,m} > B_{max,m}$ **then**
        $\lambda_{m,min} = \lambda_m$.
        $\lambda_m = (\lambda_{m,min} + \lambda_{m,max})/2$.
    else
        $\lambda_{m,max} = \lambda_m$.
        $\lambda_m = (\lambda_{m,min} + \lambda_{m,max})/2$.
    end if
end loop

The algorithm allows each base station m to update its own power allocations based on measuring the interference I_{mkn} and the latest price values t_{mkn} updated in the outer loop as shown in Algorithm 1. The interference can be measured by each AP and only the prices t_{mkn} need be exchanged leading to an essentially decentralized implementation.

As stated in [22], a convergence proof for iterative waterfilling-like algorithms is difficult in general but convergence can be guaranteed by slowing down the update:

$$t^{new}_{mkn} = \beta t^{old}_{mkn} + (1 - \beta)\hat{t}_{mkn} \tag{18}$$

for some $0 < \beta < 1$.

VI. Numerical Simulations

In this section we present the results of simulations to illustrate the efficacy of the proposed algorithm in Section V. The simulation comprises small cell network with $M = 3$ AP and $K = 10$ users distributed randomly in a $d_{area} = 500\ m$ square area. There are $N = 16$ orthogonal subchannels with the fading model as described in Section II. Each small cell has a limited transmit power of $P_{max,m} = 24\ dBm$, each subchannel has a bandwidth of 15 kHz, the user equipment noise figure is 9 dB, the noise power spectral density is -174 dBm/Hz for the additive Gaussian noise and the variance of the Log Normal Shadowing is $\sigma_s = 10$, $\beta = 0.9$, the internal wall partition loss is $W = 5\ dB$, penetration loss of an outdoor wall is set to $L = 10\ dB$ with probability 0.8 and $L = 2\ dB$ with probability 0.2, q_{mk} is fixed at 1, $\epsilon = 10^{-4}$ and the maximum number of iterations of outer loop is set to be 200. We set all the weights $\alpha_k = 1$ so all users have the same priority.

We compare the proposed algorithm (hereby denoted as IIWFB: Improved Iterative Waterfilling with Backhaul) with a baseline greedy subchannel allocation scheme which allocates each subchannel to the user and AP with the strongest channel. Each AP then performs bisection search detailed in Algorithm 2 to calculate the appropriate water level for both power and backhaul feasibility. This approach has no interference but only has a frequency reuse factor of $\boldsymbol{fr} = \frac{1}{3}$. We define frequency reuse factor as:

$$\boldsymbol{fr} = \frac{N_{active}}{NM}, \tag{19}$$

where $N_{active} = \|P_{mkn} > 0\|$ is the number of non-zero power allocations.

Figure 1 plots the WSR versus SNR for different values of $B_{max,m}$. As is clear, the proposed algorithm shows significant gains in spectral efficiency. In Figure 2 we see the achieved sum rate versus the total amount of backhaul available in the system $B_{total} = \sum_{\forall m=1}^{M} B_{max,m}$. Since the transmit power is fixed at 24 dBm each cell, as the backhaul capacity increases, the system becomes power constrained (i.e. more power is needed to achieve the total backhaul capacity). For a system in the backhaul limited regime ($B_{max,m} < 20$ (bits/s/Hz) in this simulation), the proposed algorithm achieves close to the total backhaul available. In the power-limited regime ($B_{max.m} > 20$ (bits/s/Hz) in this simulation), the algorithm behaves as the standard Improved Iterative Waterfilling algorithm. However, again, the significantly improved performance over a naive scheme is clear.

The significant increase in spectral efficiency can be attributed to the frequency reuse achieved by the algorithm - as is shown in Figure 3. Based on system fading conditions and locations of AP and users, the algorithm will find the optimum level frequency reuse.

VII. Conclusion

In this paper we studied joint user scheduling, cell association, spectrum allocation and power control for single

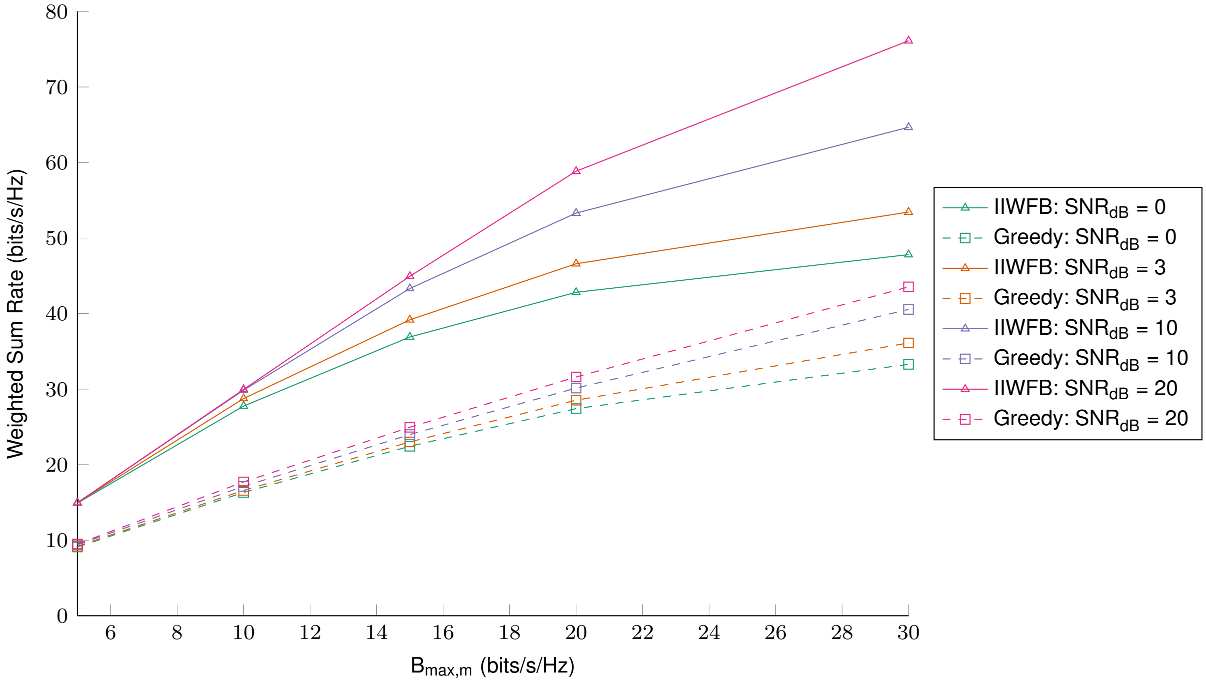

Fig. 1. Plot of Weighted Sum Rate versus $B_{max,m}$. The legend included in Figure 1 applies for all other figures.

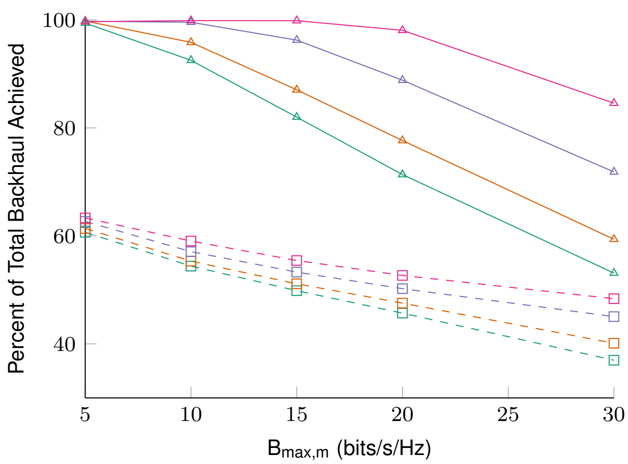

Fig. 2. Plot of Percent Backhaul Used versus $B_{max,m}$.

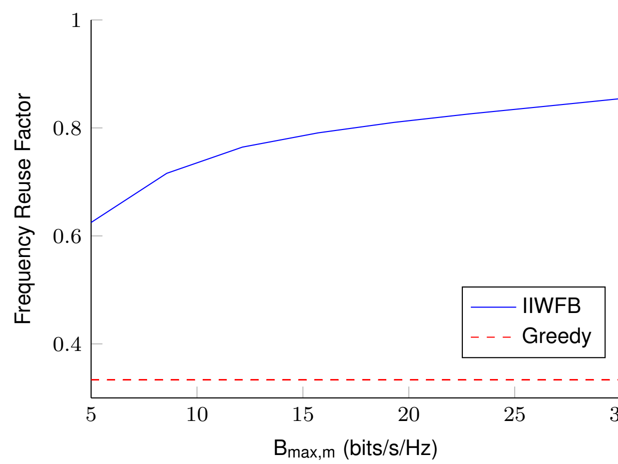

Fig. 3. Plot of Frequency Reuse Factor versus $B_{max,m}$ for $SNR_{dB} = 10$.

antenna downlink transmission with carrier aggregation. The complementary slackness property for per AP sum rate and the sum power constraint allows for a simple computation of the optimal dual variables via bisection which allows for efficient computation of locally optimum weighted sum rate utility. Further, the algorithm allows for a fairly distributed implementation with the interference measured in real time and requiring only exchange of the pricing information that sets the water level.

The dynamic frequency reuse property of this algorithm suggests that given a specific fading environment, we can use this algorithm to determine which links are weakly coupled by mutual interference. Future work should investigate efficient means of clustering cells and links to reduce overhead needed to synchronize the prices t_{mkn} and also quantify the cost of the overhead transmissions.

References

[1] A. Damnjanovic, J. Montojo, Y. Wei, T. Ji, T. Luo, M. Vajapeyam, T. Yoo, O. Song, and D. Malladi, "A survey on 3GPP heterogeneous

networks," *Wireless Communications, IEEE*, vol. 18, no. 3, p. 10–21, 2011.

[2] A. Ghosh, N. Mangalvedhe, R. Ratasuk, B. Mondal, M. Cudak, E. Visotsky, T. A. Thomas, J. G. Andrews, P. Xia, and H. S. Jo, "Heterogeneous cellular networks: From theory to practice," *Communications Magazine, IEEE*, vol. 50, no. 6, p. 54–64, 2012.

[3] R. Madan, J. Borran, A. Sampath, N. Bhushan, A. Khandekar, and T. Ji, "Cell association and interference coordination in heterogeneous LTE-A cellular networks," *IEEE Journal on Selected Areas in Communications*, vol. 28, no. 9, pp. 1479–1489, 2010.

[4] J. Andrews, "Seven ways that HetNets are a cellular paradigm shift," *IEEE Communications Magazine*, vol. 51, no. 3, pp. 136–144, 2013.

[5] J. Andrews, H. Claussen, M. Dohler, S. Rangan, and M. Reed, "Femtocells: Past, present, and future," *IEEE Journal on Selected Areas in Communications*, vol. 30, no. 3, pp. 497–508, 2012.

[6] V. Chandrasekhar, J. Andrews, and A. Gatherer, "Femtocell networks: a survey," *Communications Magazine, IEEE*, vol. 46, no. 9, p. 59–67, 2008.

[7] R. Irmer, H. Droste, P. Marsch, M. Grieger, G. Fettweis, S. Brueck, H.-P. Mayer, L. Thiele, and V. Jungnickel, "Coordinated multipoint: Concepts, performance, and field trial results," *IEEE Communications Magazine*, vol. 49, no. 2, pp. 102 –111, Feb. 2011.

[8] Y. Huang, C. W. Tan, and B. D. Rao, "Joint beamforming and power control in coordinated multicell: Max-min duality, effective network and large system transition," *arXiv:1303.2774*, Mar. 2013.

[9] A. Tolli, H. Pennanen, and P. Komulainen, "Distributed coordinated multi-cell transmission based on dual decomposition," in *IEEE Global Telecommunications Conference, 2009. GLOBECOM 2009*, 2009, pp. 1–6.

[10] P. Marsch and G. Fettweis, *On Downlink Network MIMO under a Constrained Backhaul and Imperfect Channel Knowledge*.

[11] P. Xia, H.-S. Jo, and J. Andrews, "Fundamentals of inter-cell overhead signaling in heterogeneous cellular networks," *IEEE Journal of Selected Topics in Signal Processing*, vol. 6, no. 3, pp. 257–269, 2012.

[12] F. Pantisano, M. Bennis, W. Saad, M. Debbah, and M. Latva-aho, "On the impact of heterogeneous backhauls on coordinated multipoint transmission in femtocell networks," in *2012 IEEE International Conference on Communications (ICC)*, 2012, pp. 5064–5069.

[13] Y. Cui, V. K. N. Lau, and H. Huang, "Dynamic partial cooperative MIMO system for delay-sensitive applications with limited backhaul capacity," arXiv e-print 1307.2320, Jul. 2013.

[14] A. Ghosh, R. Ratasuk, B. Mondal, N. Mangalvedhe, and T. Thomas, "LTE-advanced: next-generation wireless broadband technology [invited paper]," *IEEE Wireless Communications*, vol. 17, no. 3, pp. 10–22, 2010.

[15] X. Lin, J. G. Andrews, and A. Ghosh, "Modeling, analysis and design for carrier aggregation in heterogeneous cellular networks," arXiv e-print 1211.4041, Nov. 2012.

[16] K. Srinivas, A. Eckford, and R. Adve, "Fractional cooperation in femtocell networks," in *2012 IEEE Global Communications Conference (GLOBECOM)*, 2012, pp. 4380–4385.

[17] A. Chowdhery, W. Yu, and J. M. Cioffi, "Cooperative wireless multicell OFDMA network with backhaul capacity constraints," in *Communications (ICC), 2011 IEEE International Conference on*, 2011, p. 1–6.

[18] S. Mehryar, A. Chowdhery, and W. Yu, "Dynamic cooperation link selection for network MIMO systems with limited backhaul capacity," in *Communications (ICC), 2012 IEEE International Conference on*, 2012, p. 4410–4415.

[19] I. Marić, B. Boštjančič, and A. Goldsmith, "Resource allocation for constrained backhaul in picocell networks," in *Information Theory and Applications Workshop (ITA), 2011*, 2011, pp. 1–6.

[20] A. Agustin, J. Vidal, O. Muñoz-Medina, and J. R. Fonollosa, "Decentralized weighted sum rate maximization in MIMO-OFDMA femtocell networks," in *GLOBECOM Workshops (GC Wkshps), 2011 IEEE*, 2011, p. 270–274.

[21] C. Kim, R. Ford, Y. Qi, and S. Rangan, "Joint interference and user association optimization in cellular wireless networks," *arXiv preprint arXiv:1304.3977*, 2013.

[22] W. Yu, "Multiuser water-filling in the presence of crosstalk," in *Information Theory and Applications Workshop, 2007*, 2007, p. 414–420.

[23] "Evolved universal terrestrial radio access (e-ultra): Physical channels and modulation," *3GPP TS 36.211 v8.2.0*, 2008.

[24] F. P. Kelly, A. K. Maulloo, and D. K. Tan, "Rate control for communication networks: shadow prices, proportional fairness and stability," *Journal of the Operational Research society*, vol. 49, no. 3, p. 237–252, 1998.

[25] J. Nocedal and S. J. Wright, *Numerical optimization*. New York: Springer, 1999.

[26] S. P. Boyd and L. Vandenberghe, *Convex optimization*. Cambridge, UK; New York: Cambridge University Press, 2004.